Analysis of the radiative decays among the charmonium states

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Abstract

In this article, we study the radiative decays among the charmonium states with the heavy quark effective theory, and make predictions for the ratios among the radiative decay widths of an special multiplet to another multiplet. The predictions can be confronted with the experimental data in the future and put additional constraints in identifying the X, Y, Z charmonium-like mesons.

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1 Introduction

In recent years, the Babar, Belle, CLEO, D0, CDF and FOCUS collaborations have discovered (or confirmed) a large number of charmonium-like states and revitalized the interest in the spectroscopy of the charmonium states. Many possible assignments for those states have been suggested, such as the conventional charmonium states, the multiquark states (irrespective of the molecule type and the diquark-antidiquark type), the hybrid states, the baryonium states, the threshold effects, etc [1, 2, 3, 4, 5, 6, 7]. The main difficulties in identifying those new mesons as the excited charmonium states in that the observed masses do not fit in with the predictions of the potential models based on the confining potential. In this article, we will focus on the traditional charmonium scenario and study the radiative decays among the charmonium states with the heavy quark effective theory [8, 9, 10].

In 2003, the Belle collaboration observed the $X(3872)$ in the $J/\psi\pi^+\pi^-$ invariant mass distribution with a significance in excess of $10\,\sigma$ in the $B \to KJ/\psi\pi^+\pi^-$ decay [11], and later the $X(3872)$ was confirmed by the D0, CDF and Babar collaborations [12, 13, 14]. The decay $X(3872) \to J/\psi\gamma$ observed by the Belle and Babar collaborations [15, 16, 17] and the decay $X(3872) \to \psi'\gamma$ observed by the Babar collaboration [17] favor the even charge conjunction assignment. The di-pion spectrum also indicates that the charge conjunction is even [11], and the studies of the Belle collaboration [15] and CDF collaboration [18, 19] favor the spin-parity-charge-conjunction $J^{PC} = 1^{++}$ assignment for the $X(3872)$. However, the recent analysis of the $B \to J/\psi\omega K$ decay by the Babar collaboration indicates that the P-wave orbital angular momentum for the $J/\psi\omega$ system is more favored than the S-wave, the $X(3872)$ maybe have the spin-parity-charge-conjunction $J^{PC} = 2^{-+}$ instead of the $J^{PC} = 1^{++}$ [20]. In Ref.[21], Jia et al study the radiative decays $X(3872) \to J/\psi(\psi') + \gamma$ within several phenomenological potential models, and observe that the 2^{-+} assignment for the $X(3872)$ is highly problematic. In this article, we assume that the $X(3872)$ is the conventional charmonium $\chi_{c1}(2\mathrm{P})$ [22, 23, 24, 25, 26].

In 2005, the Belle collaboration observed the $Z(3930)$ in the $D\bar{D}$ invariant mass distribution near $3.93\,\mathrm{GeV}$ in the $\gamma\gamma$ collision with the statistical significance of $5.3\,\sigma$ [27], the mass and width are $M_{Z(3930)} = (3929 \pm 5 \pm 2)\,\mathrm{MeV}$ and $\Gamma_{Z(3930)} = (29 \pm 10 \pm 2)\,\mathrm{MeV}$, respectively. The production rate and the angular distribution favor the $\chi_{c2}(2\mathrm{P})$ assignment [27]. In the same year, the Belle collaboration observed the $X(3940)$ in the recoiling spectrum of the J/ψ in the process $e^+e^- \to J/\psi + D^*\bar{D}$ [28]. Later the Belle collaboration studied it with higher statistics and determined the mass and width $M_{X(3940)} = \left(3942^{+7}_{-6} \pm 6\right)$ MeV and $\Gamma_{X(3940)} = \left(37^{+26}_{-15} \pm 8\right)$ MeV [29]. Furthermore, they observed the $X(4160)$ in the $D^*\bar{D}^*$ invariant mass distribution in the process $e^+e^- \to J/\psi + D^*\bar{D}^*$ with a significance of $5.1\,\sigma$. The mass and width of the $X(4160)$ are $M_{X(4160)} = \left(4156^{+25}_{-20} \pm 15\right)$ MeV and $\Gamma_{X(4160)} = \left(139^{+111}_{-61} \pm 21\right)$ MeV, respectively. The observation of the dominant decay mode of the $X(3940)$ being the $D^*\bar{D}$ and the lack of evidence for the

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$D\bar{D}$ decay mode indicate that it is a good candidate for the $\eta_c(3\text{S})$ state [28, 29, 30], although the mass is lower than the prediction of the potential models [31]. The dominant decay mode of the $X(4160)$ is the $D^*\bar{D}^*$ [29], and the $D\bar{D}$ and $D\bar{D}^*$ modes have not been observed, so the $X(4160)$ may be the $\eta_c(4\text{S})$ or $\chi_{c0}(3\text{P})$ candidate [32].

Also in 2005, the Babar collaboration observed the $Y(4260)$ in the $J/\psi\pi^+\pi^-$ invariant mass distribution in the initial state radiation (ISR) process $e^+e^- \to \gamma_{\rm ISR}J/\psi\pi^+\pi^-$ [33]. It was confirmed by the CLEO collaboration [34] and Belle collaboration [35], and in the Belle data there is also a relative broad structure $Y(4008)$ with the mass $M_{Y(4008)} = \left(4008 \pm 40^{+114}_{-28}\right)$ MeV and width $\Gamma_{Y(4008)} = (226 \pm 44 \pm 87)$ MeV, respectively. In Ref.[36], Llanes-Estrada suggests that the $Y(4260)$ may be the $\psi(4\text{S})$ charmonium state and displaces the $\psi(4415)$.

In 2006, the Babar collaboration observed the structure $Y(4320)$ in the initial state radiation process $e^+e^- \to \gamma_{\rm ISR}\psi'\pi^+\pi^-$ with the mass $M_{Y(4320)} = (4324 \pm 24)$ MeV and width $\Gamma_{Y(4320)} = (172 \pm 33)$ MeV, respectively [37], and later the Belle collaboration observed two relative narrow resonant structures $Y(4360)$ and $Y(4660)$ with the masses $M_{Y(4360)} = (4361 \pm 9 \pm 9)$ MeV, $M_{Y(4660)} = (4664 \pm 11 \pm 5)$ MeV and the widths $\Gamma_{Y(4360)} = (74 \pm 15 \pm 10)$ MeV, $\Gamma_{Y(4660)} = (48 \pm 15 \pm 3)$ MeV, respectively [38]. In 2008, the Belle collaboration observed the $Y(4630)$ as a threshold enhancement in the $\Lambda_c^+\Lambda_c^-$ invariant mass distribution in the initial state radiation process $e^+e^- \to \gamma_{\rm ISR}\Lambda_c^+\Lambda_c^-$ [39]. The mass and width are $M_{Y(4630)} = \left(4634^{+8+5}_{-7-8}\right)$ MeV and $\Gamma_{Y(4630)} = \left(92^{+40+10}_{-24-21}\right)$ MeV, respectively, which are roughly in agreement with that of the $Y(4660)$. In Ref.[26], Li and Chao calculate the mass spectrum of the charmonium states based on the screened potential, and observe that the $Z(3930)$ agrees with the $\chi_{c2}(2\text{P})$, the $\psi(4415)$ is compatible with the $\psi(5\text{S})$ rather than the $\psi(4\text{S})$, the $Y(4260)$, $Y(4360)$ and $Y(4660)$ may be the $\psi(4\text{S})$, $\psi(3\text{D})$ and $\psi(6\text{S})$ respectively, and the $X(3940)$ and $X(4160)$ may be the $\eta_c(3\text{S})$ and $\chi_{c0}(3\text{P})$ respectively. In Ref.[40], Ding et al identify the $Y(4360)$ and $Y(4660)$ as the 3^3D_1 and 5^3S_1 charmonium states respectively (Badalian et al share the same opinion [41]), and evaluate the e^+e^- leptonic widths, $E1$ transitions, $M1$ transitions and the open flavor strong decays.

In 2009, the CDF collaboration observed a narrow structure $Y(4140)$ near the $J/\psi\phi$ threshold with a statistical significance in excess of $3.8\,\sigma$ in the exclusive $B \to J/\psi\phi K$ decays produced in $\bar{p}p$ collisions [42]. The mass and width are $M_{Y(4140)} = (4143.0 \pm 2.9 \pm 1.2)$ MeV and $\Gamma_{Y(4140)} = \left(11.7^{+8.3}_{-5.0} \pm 3.7\right)$ MeV, respectively [42]. The $Y(4140)$ is very similar to the charmonium-like state $Y(3940)$, which was observed by both the Belle and Babar collaborations near the $J/\psi\omega$ threshold in the exclusive $B \to J/\psi\omega K$ decays [43, 44]. The mass and width are $M_{Y(3940)} = (3943 \pm 11 \pm 13)$ MeV and $\Gamma_{Y(3940)} = (87 \pm 22 \pm 26)$ MeV respectively from the Belle collaboration [43] and $M_{Y(3940)} = \left(3914.6^{+3.8}_{-3.4} \pm 2.0\right)$ MeV and $\Gamma_{Y(3940)} = \left(34^{+12}_{-8} \pm 5\right)$ MeV respectively from the Babar collaboration [44]. In 2009, the Belle collaboration reported the observation of a significant enhancement with the mass $(3915 \pm 3 \pm 2)$ MeV and total width $(17 \pm 10 \pm 3)$ MeV respectively in the process $\gamma\gamma \to \omega J/\psi$ [45], these values are consistent with that of the $Y(3940)$. The updated values of the mass $\left(3919.1^{+3.8}_{-3.5} \pm 2.0\right)$ MeV and total width $\left(31^{+10}_{-8} \pm 5\right)$ MeV from the Babar collaboration are also consistent with the old ones [46]. The Belle collaboration measured the process $\gamma\gamma \to \phi J/\psi$ for the $J/\psi\phi$ invariant mass distributions between the threshold and 5 GeV, and observed a narrow peak $X(4350)$ with a significance of $3.2\,\sigma$ [47]. The mass and width are $M_{X(4350)} = \left(4350.6^{+4.6}_{-5.1} \pm 0.7\right)$ MeV and $\Gamma_{X(4350)} = \left(13.3^{+17.9}_{-9.1} \pm 4.1\right)$ MeV, respectively; and no signal for the $Y(4140) \to J/\psi\phi$ structure was observed [47]. It is difficult to identify the $Y(3940)$, $Y(4140)$ and $X(4350)$ as the conventional charmonium states [48].

In Table 1, we list the experimental values of the charmonium states with the possible identifications compared with the theoretical predictions [26, 31, 49]. We do not mean that such assignments are correct and exclude other possibilities, and just take it for granted for the moment and study the radiative decays among the charmonium states with the heavy quark effective theory [8, 9, 10], which have been applied to identify the excited D_s and D mesons, such as the $D_s(3040)$, $D_s(2700)$, $D_s(2860)$, $D(2550)$, $D(2600)$, $D(2750)$ and $D(2760)$ [50, 51, 52, 53, 54].

The article is arranged as follows: we study the radiative decays among the charmonium states

	State	Experiment [49]	SP [26]	NRP [31]	GI [31]
1S	$J/\psi(1^3\mathrm{S}_1)$	3096.916	3097	3090	3098
	$\eta_c(1^1\mathrm{S}_0)$	2980.3	2979	2982	2975
2S	$\psi'(2^3\mathrm{S}_1)$	3686.09	3673	3672	3676
	$\eta_c'(2^1\mathrm{S}_0)$	3637	3623	3630	3623
3S	$\psi(3^3\mathrm{S}_1)$	4039 [$\psi(4040)$]	4022	4072	4100
	$\eta_c(3^1\mathrm{S}_0)$	**?** 3942 [$X(3940)$]	3991	4043	4064
4S	$\psi(4^3\mathrm{S}_1)$	**?** 4263 [$Y(4260)$]	4273	4406	4450
	$\eta_c(4^1\mathrm{S}_0)$		4250	4384	4425
5S	$\psi(5^3\mathrm{S}_1)$	**?** 4421 [$\psi(4415)$]	4463		
	$\eta_c(5^1\mathrm{S}_0)$		4446		
6S	$\psi(6^3\mathrm{S}_1)$	**?** 4664 [$Y(4660)$]	4608		
	$\eta_c(6^1\mathrm{S}_0)$		4595		
1P	$\chi_2(1^3\mathrm{P}_2)$	3556.20	3554	3556	3550
	$\chi_1(1^3\mathrm{P}_1)$	3510.66	3510	3505	3510
	$\chi_0(1^3\mathrm{P}_0)$	3414.75	3433	3424	3445
	$h_c(1^1\mathrm{P}_1)$	3525.42	3519	3516	3517
2P	$\chi_2(2^3\mathrm{P}_2)$	3929 [$Z(3930)$]	3937	3972	3979
	$\chi_1(2^3\mathrm{P}_1)$	**?** 3871.56 [$X(3872)$]	3901	3925	3953
	$\chi_0(2^3\mathrm{P}_0)$		3842	3852	3916
	$h_c(2^1\mathrm{P}_1)$		3908	3934	3956
3P	$\chi_2(3^3\mathrm{P}_2)$		4208	4317	4337
	$\chi_1(3^3\mathrm{P}_1)$		4178	4271	4317
	$\chi_0(3^3\mathrm{P}_0)$	**?** 4156 [$X(4160)$]	4131	4202	4292
	$h_c(3^1\mathrm{P}_1)$		4184	4279	4318
1D	$\psi_3(1^3\mathrm{D}_3)$		3799	3806	3849
	$\psi_2(1^3\mathrm{D}_2)$		3798	3800	3838
	$\psi(1^3\mathrm{D}_1)$	3772.92 [$\psi(3770)$]	3787	3785	3819
	$\eta_{c2}(1^1\mathrm{D}_2)$		3796	3799	3837
2D	$\psi_3(2^3\mathrm{D}_3)$		4103	4167	4217
	$\psi_2(2^3\mathrm{D}_2)$		4100	4158	4208
	$\psi(2^3\mathrm{D}_1)$	4153 [$\psi(4160)$]	4089	4142	4194
	$\eta_{c2}(2^1\mathrm{D}_2)$		4099	4158	4208
3D	$\psi_3(3^3\mathrm{D}_3)$		4331		
	$\psi_2(3^3\mathrm{D}_2)$		4327		
	$\psi(3^3\mathrm{D}_1)$	**?** 4361 [$Y(4360)$]	4317		
	$\eta_{c2}(3^1\mathrm{D}_2)$		4326		

Table 1: Experimental and theoretical mass spectrum of the charmonium states. The SP, NRP and GI denote the screened potential model, the non-relativistic potential model and the Godfrey-Isgur relativized potential model, respectively.

with the heavy quark effective theory in Sect.2; in Sect.3, we present the numerical results and discussions; and Sect.4 is reserved for our conclusions.

2 The radiative decays with the heavy quark effective theory

The charmonium states can be classified according to the notation $n^{2s+1}L_j$, where the n is the radial quantum number, the L is the orbital angular momentum, the s is the spin, and the j is the total angular momentum. They have the parity and charge conjugation $P=(-1)^{L+1}$ and $C=(-1)^{L+s}$, respectively. The states have the same radial quantum number n and orbital momentum L can be expressed by the superfields J, J^μ, $J^{\mu\nu}$, etc [55],

$$
\begin{aligned}
J &= \frac{1+\not{v}}{2}\left\{\psi_\mu\gamma^\mu-\eta_c\gamma_5\right\}\frac{1-\not{v}}{2}\,, \\
J^\mu &= \frac{1+\not{v}}{2}\left\{\chi_2^{\mu\nu}\gamma_\nu+\frac{1}{\sqrt{2}}\epsilon^{\mu\alpha\beta\lambda}v_\alpha\gamma_\beta\chi^1_\lambda+\frac{1}{\sqrt{3}}\left(\gamma^\mu-v^\mu\right)\chi_0+h_c^\mu\gamma_5\right\}\frac{1-\not{v}}{2}\,, \\
J^{\mu\nu} &= \frac{1+\not{v}}{2}\left\{\chi_3^{\mu\nu\alpha}\gamma_\alpha+\frac{1}{\sqrt{6}}\left[\epsilon^{\mu\alpha\beta\lambda}v_\alpha\gamma_\beta g^{\tau\nu}+\epsilon^{\nu\alpha\beta\lambda}v_\alpha\gamma_\beta g^{\tau\mu}\right]\chi^2_{\tau\lambda}+\right. \\
&\left.\left[\sqrt{\frac{3}{20}}\left[\left(\gamma^\mu-v^\mu\right)g^{\nu\alpha}+\left(\gamma^\nu-v^\nu\right)g^{\mu\alpha}\right]-\frac{1}{\sqrt{15}}\left(g^{\mu\nu}-v^\mu v^\nu\right)\gamma^\alpha\right]\chi^1_\alpha+\eta^{\mu\nu}_{c2}\gamma_5\right\}\frac{1-\not{v}}{2}\,,
\end{aligned}
\tag{1}
$$

where the v^μ denotes the four-velocity associated to the superfields, the fields $\chi^3_{\mu\nu\alpha}$, $\chi^2_{\mu\nu}$, χ^1_μ, χ^0, $\eta^{\mu\nu}_{c2}$ have the total angular momentum $j=3,2,1,0,2$ respectively, and belong to different multiplets. The fields in a definite superfield have the same n, and form a multiplet. We multiply the charmonium fields $\chi^3_{\mu\nu\alpha}$, $\chi^2_{\mu\nu}$, χ^1_μ, χ^0, $\eta^{\mu\nu}_{c2}$, $\cdots$ with a factor $\sqrt{M_\chi}$, $\sqrt{M_\eta}$, $\cdots$, and they have dimension of mass $\frac{3}{2}$. The superfields $J^{\mu_1\cdots\mu_L}$ have the following properties under the parity, charge conjunction, heavy quark spin transformations,

$$
\begin{aligned}
J^{\mu_1\cdots\mu_L} &\xrightarrow{P} \gamma^0 J_{\mu_1\cdots\mu_L}\gamma^0\,, \\
J^{\mu_1\cdots\mu_L} &\xrightarrow{C} (-1)^{L+1}C[J_{\mu_1\cdots\mu_L}]^T C\,, \\
J^{\mu_1\cdots\mu_L} &\xrightarrow{S} SJ_{\mu_1\cdots\mu_L}S'^{\dagger}\,, \\
v^\mu &\xrightarrow{P} v_\mu\,,
\end{aligned}
\tag{2}
$$

where $S,S'\in SU(2)$ heavy quark spin symmetry groups, and $[S,\not{v}]=[S',\not{v}]=0$.

The radiative transitions between the m and n charmonium states can be described by the following Lagrangians,

$$
\begin{aligned}
\mathcal{L}_{SS} &= \sum_{m,n}\delta(m,n)\mathrm{Tr}\left[\bar{J}(m)\sigma_{\mu\nu}J(n)\right]F^{\mu\nu}\,, \\
\mathcal{L}_{SP} &= \sum_{m,n}\delta(m,n)\mathrm{Tr}\left[\bar{J}(m)J_\mu(n)+\bar{J}_\mu(n)J(m)\right]V^\mu\,, \\
\mathcal{L}_{PD} &= \sum_{m,n}\delta(m,n)\mathrm{Tr}\left[\bar{J}_{\mu\nu}(m)J^\nu(n)+\bar{J}^\nu(n)J_{\mu\nu}(m)\right]V^\mu\,,
\end{aligned}
\tag{3}
$$

where $\bar{J}_{\mu_1\ldots\mu_L}=\gamma^0J^\dagger_{\mu_1\ldots\mu_L}\gamma^0$, $V^\mu=F^{\mu\nu}v_\nu$, the $F^{\mu\nu}$ is the electromagnetic tensor, and the $\delta(m,n)$ are the coupling constants, which have different values in the Lagrangians $\mathcal{L}_{SS}$, $\mathcal{L}_{SP}$, $\mathcal{L}_{PD}$, we use the same notation for simplicity. The Lagrangians $\mathcal{L}_{SP}$ and $\mathcal{L}_{PD}$ preserve parity, charge

conjugation, gauge invariance and heavy quark spin symmetry, while the Lagrangian $\mathcal{L}_{SS}$ violates the heavy quark symmetry. There are two tensors $\sigma_{\mu\nu}\widetilde{F}^{\mu\nu}(=\frac{1}{2}\epsilon^{\mu\nu\alpha\beta}\sigma_{\mu\nu}F_{\alpha\beta})$ and $\sigma_{\mu\nu}F^{\mu\nu}$ can be used to construct the spin-breaking Lagrangian $\mathcal{L}_{SS}$. They both have the terms $\vec{\sigma}\cdot\vec{E}$ and $\vec{\sigma}\cdot\vec{B}$, and can turn into each other with the interchange $\vec{E}\leftrightarrow\vec{B}$, we choose the parity-conserving structure $\sigma_{\mu\nu}F^{\mu\nu}$ in constructing the spin-breaking Lagrangian. We take the Lagrangian $\mathcal{L}_{SP}$ from Ref.[8] and construct the Lagrangians $\mathcal{L}_{SS}$ and $\mathcal{L}_{PD}$, and later Dr. F. De Fazio draws my attention to Ref.[56], where the Lagrangian $\mathcal{L}_{PD}$ is introduced for the first time.

From the heavy quark effective Lagrangians $\mathcal{L}_{SS}$, $\mathcal{L}_{SP}$ and $\mathcal{L}_{PD}$, we can obtain the radiative decay widths Γ,

$$\Gamma = \frac{1}{2j+1}\sum\frac{k_\gamma}{8\pi M_i^2}|T|^2\,, \tag{4}$$

where the T denotes the scattering amplitude, the k_γ is the momentum of the final states in the center of mass coordinate, the $\sum$ denotes the sum of all the polarization vectors, the j is the total angular momentum of the initial state, and the M_i is the mass of the initial state. Cho and Wise study the radiative decays of the heavy quarkonia using the multipole expansion and heavy quark symmetry [57].

3 Numerical Results

We calculate the radiative decay widths Γ using the FeynCalc to carry out the sum of all the polarization vectors. In calculations, the masses of the charmonium states are taken as the experimental values from the Review of Particle Physics [49], see Table 1; for the unobserved charmonium states, we take the values from the screened potential model [26], and assume that the charmonium states $\psi(5^3\mathrm{S}_1)$ and $\eta_c(5^1\mathrm{S}_0)$ have degenerate masses. In this article, we identify the $\psi(4415)$ as the $\psi(5^3\mathrm{S}_1)$ charmonium state, and expect that the $\eta_c(5^1\mathrm{S}_0)$ charmonium state has slightly smaller mass. The predictions of the screened potential model are $M_{\psi(4\mathrm{S})}=4463\,\mathrm{MeV}$ and $M_{\eta_c(4\mathrm{S})}=4446\,\mathrm{MeV}$, respectively, so we take the approximation $M_{\psi(4\mathrm{S})}=M_{\eta_c(4\mathrm{S})}=4421\,\mathrm{MeV}$.

The numerical values of the radiative decay widths are presented in Tables 2-4, where we retain the unknown coupling constants $\delta(m,n)$ among the multiplets of the radial quantum numbers m and n. In general, we expect fitting the parameters $\delta(m,n)$ to the precise experimental data, however, in the present time the experimental data are far from enough. In Tables 4-6, we present the ratios of the radiative decay widths among the charmonium states.

The radiative decay widths listed in the Review of Particle Physics are

$$\begin{aligned}
\Gamma(\psi(2\mathrm{S})\to\eta_c(1\mathrm{S})\gamma) &= 1.0336\,\mathrm{KeV}\,,\\
\Gamma(\psi(2\mathrm{S})\to\chi_{c0}(1\mathrm{P})\gamma) &= 29.2448\,\mathrm{KeV}\,,\\
\Gamma(\psi(2\mathrm{S})\to\chi_{c1}(1\mathrm{P})\gamma) &= 27.968\,\mathrm{KeV}\,,\\
\Gamma(\psi(2\mathrm{S})\to\chi_{c2}(1\mathrm{P})\gamma) &= 26.5696\,\mathrm{KeV}\,,\\
\Gamma(\chi_{c0}(1\mathrm{P})\to J/\psi\gamma) &= 119.48\,\mathrm{KeV}\,,\\
\Gamma(\chi_{c1}(1\mathrm{P})\to J/\psi\gamma) &= 295.84\,\mathrm{KeV}\,,\\
\Gamma(\chi_{c2}(1\mathrm{P})\to J/\psi\gamma) &= 384.15\,\mathrm{KeV}\,,\\
\Gamma(\psi(3770)\to\chi_{c0}(1\mathrm{P})\gamma) &= 199.29\,\mathrm{KeV}\,,\\
\Gamma(\psi(3770)\to\chi_{c1}(1\mathrm{P})\gamma) &= 79.17\,\mathrm{KeV}\,,\\
\Gamma(\psi(3770)\to\chi_{c2}(1\mathrm{P})\gamma) &< 24.57\,\mathrm{KeV}\,,\\
\Gamma(\psi(4040)\to\chi_{c1}(1\mathrm{P})\gamma) &< 0.88\,\mathrm{MeV}\,,\\
\Gamma(\psi(4040)\to\chi_{c2}(1\mathrm{P})\gamma) &< 1.36\,\mathrm{MeV}\,,
\end{aligned} \tag{5}$$

where we have neglected the uncertainties [49]. From those radiative decay widths, we can obtain the following ratios,

$$
\begin{aligned}
\frac{\Gamma(\psi(2\mathrm{S})\to\chi_{c0}(1\mathrm{P})\gamma)}{\Gamma(\psi(2\mathrm{S})\to\chi_{c1}(1\mathrm{P})\gamma)} &= 1.046\,(1.151)\,,\\
\frac{\Gamma(\psi(2\mathrm{S})\to\chi_{c0}(1\mathrm{P})\gamma)}{\Gamma(\psi(2\mathrm{S})\to\chi_{c2}(1\mathrm{P})\gamma)} &= 1.101\,(1.649)\,,\\
\frac{\Gamma(\chi_{c0}(1\mathrm{P})\to J/\psi\gamma)}{\Gamma(\chi_{c2}(1\mathrm{P})\to J/\psi\gamma)} &= 0.311\,(0.363)\,,\\
\frac{\Gamma(\chi_{c1}(1\mathrm{P})\to J/\psi\gamma)}{\Gamma(\chi_{c2}(1\mathrm{P})\to J/\psi\gamma)} &= 0.770\,(0.756)\,,\\
\frac{\Gamma(\psi(3770)\to\chi_{c0}(1\mathrm{P})\gamma)}{\Gamma(\psi(3770)\to\chi_{c1}(1\mathrm{P})\gamma)} &= 2.517\,(3.167)\,,\\
\frac{\Gamma(\psi(3770)\to\chi_{c0}(1\mathrm{P})\gamma)}{\Gamma(\psi(3770)\to\chi_{c2}(1\mathrm{P})\gamma)} &> 8.111\,(82.02)\,,
\end{aligned}
\tag{6}
$$

where the values in the bracket are the theoretical calculations based on the heavy quark effective theory. The agreements between the experimental data and the theoretical calculations are rather good, and the heavy quark effective theory works rather well. The ratios presented in Tables 4-6 can be confronted with the experimental data in the future at the BESIII, KEK-B, RHIC, $\bar{\mathrm{P}}$ANDA and LHCb, and put additional constraints in identifying the X, Y, Z charmonium-like mesons.

There is a relative P-wave between the final-state charmonium and the photon, the radiative decay widths $\Gamma\propto k_\gamma^3$, where $k_\gamma=\frac{M_i^2-M_f^2}{2M_i}$, the M_i and M_f denote the masses of the initial and final charmonium states respectively. The numerical values of the decay widths shown in Tables 2-4, where the effective coupling constants $\delta(m,n)$ have been factorized out, reflect the corresponding processes are facilitated or suppressed in the phase-space. If the energy gap M_i-M_f is small (or large), small variations of the masses M_i and M_f can (or cannot) lead to remarkable changes for the decay width. For example, we plot the radiative decay widths $\Gamma(X(3872)\to J/\psi\gamma)$ and $\Gamma(X(3872)\to\psi'\gamma)$ versus the mass M_X in Fig.1.

From the experimental data of the Babar collaboration $\mathrm{Br}(B^+\to X(3872)K^+)\times\mathrm{Br}(X(3872)\to J/\psi\gamma)=(2.8\pm0.8\pm0.2)\times10^{-6}$ and $\mathrm{Br}(B^+\to X(3872)K^+)\times\mathrm{Br}(X(3872)\to\psi'\gamma)=(9.9\pm2.9\pm0.6)\times10^{-6}$ [17], we can obtain the central value of the ratio $\frac{\Gamma(X(3872)\to J/\psi\gamma)}{\Gamma(X(3872)\to\psi'\gamma)}=0.283$, which means $\frac{\delta^2(2,1)}{\delta^2(2,2)}=0.00576$. The large hierarchy $\delta(2,1)\ll\delta(2,2)$ is compatible with the phenomenological expectation that the $\chi_{c1}(2\mathrm{P})$ state potentially decays to the $\psi'\gamma$ rather than to the $J/\psi\gamma$,

The E_1 and M_1 transitions among the charmonium states are usually calculated by the formula [58, 59, 60, 61],

$$
\begin{aligned}
\Gamma_{E1}\left(\mathrm{n}^{2s+1}L_j\to\mathrm{n}'^{2s'+1}L'_{j'}+\gamma\right) &= \frac{4}{3}e_c^2\alpha E_\gamma^3\frac{E_f}{M_i}\delta_{ss'}C_{fi}\mid\langle\mathrm{n}'^{2s'+1}L'_{j'}\mid r\mid\mathrm{n}^{2s+1}L_j\rangle\mid^2\,,\\
\Gamma_{M1}\left(\mathrm{n}^{2s+1}L_j\to\mathrm{n}'^{2s'+1}L'_{j'}+\gamma\right) &= \frac{4}{3}e_c^2\frac{\alpha}{m_c^2}E_\gamma^3\frac{E_f}{M_i}\frac{2j'+1}{2L+1}\delta_{LL'}\delta_{ss'\pm1}\mid\langle\mathrm{n}'^{2s'+1}L'_{j'}\mid\mathrm{n}^{2s+1}L_j\rangle\mid^2\,,
\end{aligned}
\tag{7}
$$

where the E_γ is the photon energy, the E_f is the energy of final state charmonium, the M_i is the mass of the initial state charmonium, and the angular matrix factor C_{fi} is

$$
C_{fi}=\max(L,L')(2j'+1)\left\{\begin{array}{ccc}L' & J' & s\\ J & L & 1\end{array}\right\}^2\,. \tag{8}
$$

The values of the matrix elements $\langle\mathrm{n}'^{2s'+1}L'_{j'}\mid r\mid\mathrm{n}^{2s+1}L_j\rangle$ and $\langle\mathrm{n}'^{2s'+1}L'_{j'}\mid\mathrm{n}^{2s+1}L_j\rangle$ depend on the details of the wave-functions which are evaluated using a special potential model, for example,

	$\Gamma(\psi \to \chi_{c2}\gamma)$	$\Gamma(\psi \to \chi_{c1}\gamma)$	$\Gamma(\psi \to \chi_{c0}\gamma)$	$\Gamma(\eta_c \to h_c\gamma)$
2S $\to$ 1P	3.546	5.082	5.849	4.092
3S $\to$ 1P	146.4	112.1	57.15	174.9
3S $\to$ 2P	2.198	4.488	2.388	0.122
4S $\to$ 1P	405.8	287.4	126.3	768.9
4S $\to$ 2P	53.97	50.43	20.43	103.5
4S $\to$ 3P	0.285	0.620	0.407	0.880
5S $\to$ 1P	685.4	471.2	193.7	1323
5S $\to$ 2P	158.3	128.3	48.69	317.5
5S $\to$ 3P	15.13	13.26	5.647	36.96
6S $\to$ 1P	1283	858.8	328.0	2062
6S $\to$ 2P	466.5	341.7	122.7	695.3
6S $\to$ 3P	130.6	93.49	34.93	175.4
1D $\to$ 1P	0.258	6.691	21.19	
2D $\to$ 1P	4.205	78.91	147.5	
2D $\to$ 2P	0.288	8.305	14.63	
3D $\to$ 1P	9.075	164.7	277.1	
3D $\to$ 2P	1.821	38.97	60.39	
3D $\to$ 3P	0.097	2.438	4.506	

Table 2: The radiative decay widths of the S-wave and D-wave charmonium states to the P-wave charmonium states. The unit is $10^{-4}\delta^2(m,n)$.

the Cornell potential model, the logarithmic potential model, the power-law potential model, the QCD-motivated potential model [62], the relativized Godfrey-Isgur model, the non-relativistic potential model [31], the screened potential model [26], the relativistic quark model based on a quasipotential approach in QCD [63, 64], etc. All predictions should be confronted with the experimental data. In this article, we intend to make estimations based on the heavy quark effective theory, and prefer the ratios among the radiative decay widths of an special multiplet to another multiplet, where the unknown parameters $\delta(m,n)$ are canceled out with each other.

4 Conclusion

In this article, we study the radiative decays among the charmonium states with the heavy quark effective theory, and make predictions for ratios among the radiative decay widths of an special multiplet to another multiplet, where the unknown couple constants $\delta(m,n)$ are canceled out with

	$\Gamma(\chi_{c2} \to \psi\gamma)$	$\Gamma(\chi_{c1} \to \psi\gamma)$	$\Gamma(\chi_{c0} \to \psi\gamma)$	$\Gamma(h_c \to \eta_c\gamma)$
1P $\to$ 1S	73.70	55.69	26.78	114.1
2P $\to$ 1S	350.4	294.9	260.5	442.2
2P $\to$ 2S	13.00	6.000	3.628	17.68
3P $\to$ 1S	720.8	686.1	624.0	827.2
3P $\to$ 2S	109.6	93.62	82.01	123.2
3P $\to$ 3S	4.628	2.621	1.583	12.97

Table 3: The radiative decay widths of the P-wave charmonium states to the S-wave charmonium states. The unit is $10^{-4}\delta^2(m,n)$.

	$\Gamma(\psi \to \eta_c \gamma)$	$\Gamma(\eta_c \to \psi\gamma)$	$\frac{\Gamma(\eta_c \to \psi\gamma)}{\Gamma(\psi \to \eta_c \gamma)}$
$2S \to 1S$	0.089	0.136	1.519
$3S \to 1S$	0.244	0.430	1.762
$3S \to 2S$	0.021	0.018	0.848
$4S \to 1S$	0.384	0.919	2.392
$4S \to 2S$	0.071	0.161	2.281
$4S \to 3S$	0.012	0.011	0.911
$5S \to 1S$	0.502	1.273	2.536
$5S \to 2S$	0.127	0.325	2.550
$5S \to 3S$	0.035	0.057	1.614
$5S \to 4S$	0.002	0.005	2.384
$6S \to 1S$	0.712	1.692	2.375
$6S \to 2S$	0.253	0.561	2.221
$6S \to 3S$	0.106	0.160	1.505
$6S \to 4S$	0.024	0.039	1.616
$6S \to 5S$	0.005	0.006	1.143

Table 4: The ratios of the radiative decay widths of the S-wave charmonium states to the S-wave charmonium states. The unit of the widths is $\delta^2(m,n)$.

	$\Gamma(\psi \to \chi_{c1}\gamma)$	$\Gamma(\psi \to \chi_{c0}\gamma)$	$\Gamma(\eta_c \to h_c\gamma)$
$2S \to 1P$	1.433	1.649	1.154
$\widehat{2S \to 1P}$	1.053	1.101	
$3S \to 1P$	0.766	0.390	1.195
$3S \to 2P$	2.042	1.087	0.056
$4S \to 1P$	0.708	0.311	1.895
$4S \to 2P$	0.934	0.379	1.918
$4S \to 3P$	2.176	1.428	3.090
$5S \to 1P$	0.688	0.283	1.931
$5S \to 2P$	0.810	0.308	2.006
$5S \to 3P$	0.876	0.373	2.443
$6S \to 1P$	0.669	0.256	1.607
$6S \to 2P$	0.732	0.263	1.490
$6S \to 3P$	0.716	0.267	1.343
$1D \to 1P$	25.91	82.02	
$\widehat{1D \to 1P}$	> 3.222	> 8.111	
$2D \to 1P$	18.77	35.08	
$2D \to 2P$	28.85	50.81	
$3D \to 1P$	18.15	30.54	
$3D \to 2P$	21.40	33.17	
$3D \to 3P$	25.27	46.71	

Table 5: The ratios of the radiative decay widths of the S-wave and D-wave charmonium states to the P-wave charmonium states. There we normalize $\Gamma_{\psi \to \chi_{c2}\gamma} = 1$, the wide-hat denotes the experiential values.

	$\Gamma(\chi_{c1} \to \psi\gamma)$	$\Gamma(\chi_{c0} \to \psi\gamma)$	$\Gamma(h_c \to \eta_c\gamma)$
1P $\to$ 1S	0.756	0.363	1.549
$\widehat{\text{1P} \to \text{1S}}$	0.770	0.311	
2P $\to$ 1S	0.842	0.744	1.262
2P $\to$ 2S	0.462	0.279	1.360
3P $\to$ 1S	0.952	0.866	1.148
3P $\to$ 2S	0.854	0.748	1.124
3P $\to$ 3S	0.566	0.342	2.803

Table 6: The ratios of the radiative decay widths of the P-wave charmonium states to the S-wave charmonium states. There we normalize $\Gamma_{\chi_{c2}\to\psi\gamma} = 1$, the wide-hat denotes the experiential values.

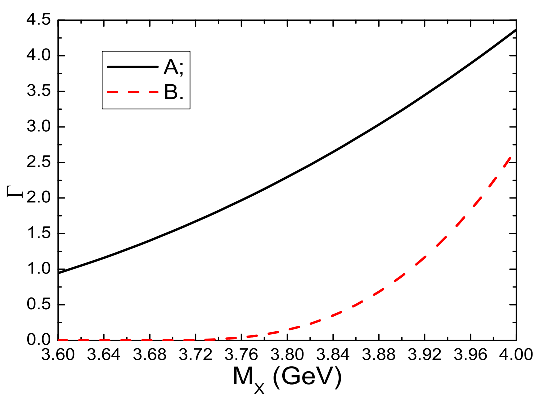

Figure 1: The radiative decay widths of the $X(3872)$ versus its mass, the A and B denote the processes $X(3872) \to J/\psi\gamma$ and $X(3872) \to \psi'\gamma$, respectively; the units of the decay widths are $10^{-2}\delta^2(2,1)$ and $10^{-3}\delta^2(2,2)$, respectively, $\delta(2,2) \gg \delta(2,1)$.

each other. The predictions can be confronted with the experimental data in the future at the BESIII, KEK-B, RHIC, $\bar{\mathrm{P}}$ANDA and LHCb, and put additional constraints in identifying the X, Y, Z charmonium-like mesons.

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